

1.75

2. Equivalent Static Load Method

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طريقة الحمل الإستاتيكي المكافئ

هي إحدى طرق التعامل مع أحمال الصدم :

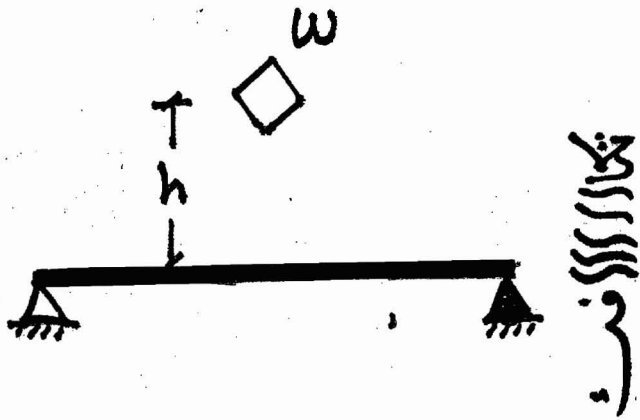
لكل طريقة مجموعة شروط للوصول لها

شروط النظرية :

1. مادة من منطقة مرونة .

2. إهمال أى فواقد للطاقة (الطاقة الداخلية = الخارجية) .

3. إهمال وزن الكتلة المعرضة للصدم بالنسبة لحمل الصدم .



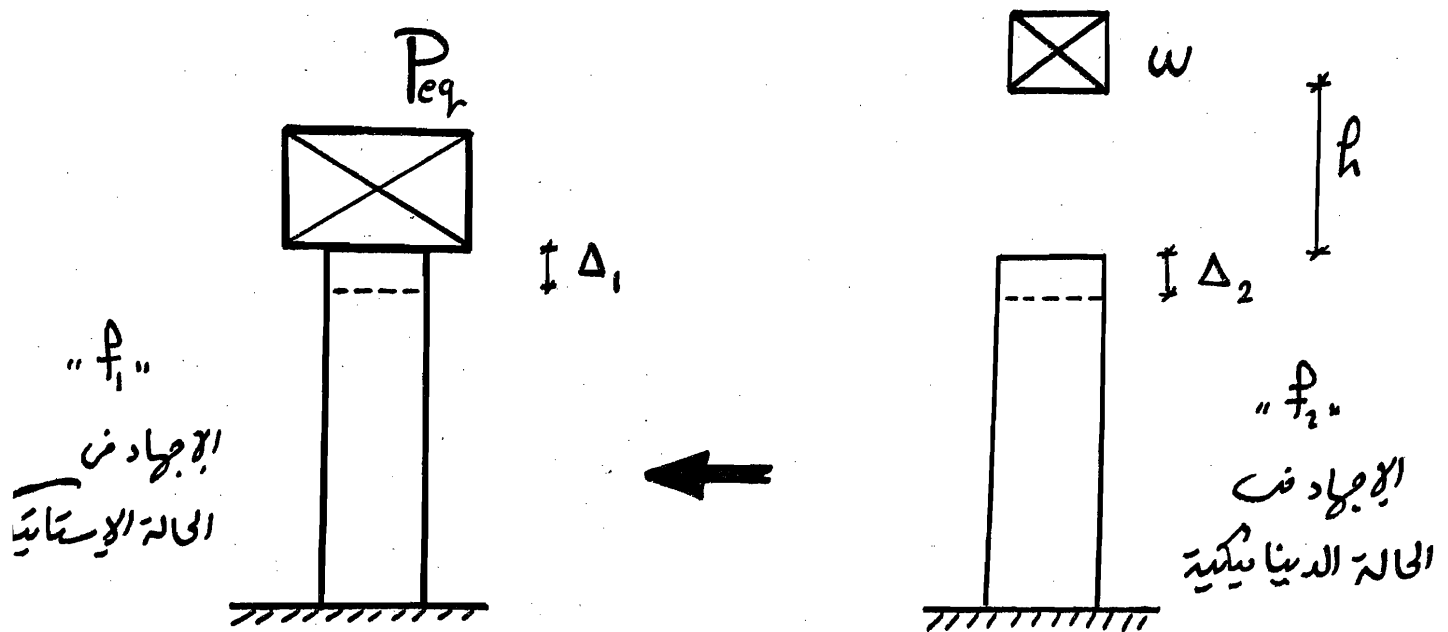
د،

طريقة الحمل الإستاتيكي المكافئ

Equivalent Static Load (E.S.L)

* الحمل الإستاتيكي المكافئ :

هو الحمل الإستاتيكي الذي يسبب إجهاد وإفعال يكافئ الإجهاد والإفعال الناتج من لصدم .



$$\Delta_1 = \Delta_2$$

$$f_1 = f_2$$

∴ $P_{eq} =$ الحمل الإستاتيكي المكافئ

$w =$ حمل لصدم

$h =$ إرتفاع إسقوط الحمل (w)

المعادلة العامة :

$$W (h + \Delta) = \frac{1}{2} P_{eq} \Delta$$

لحساب

→ للأجسام المساقطة تحت تأثير وزنها ←

- خطوات الحل :

(1) أوجد الجاهل بدلالة " P_{eq} " .

(2) عوّض من المعادلة العامة .

(3) أوجد P_{eq} ومنها Δ .

هذه النظرية لها مجموعة شروط "Assumption" :

1. المادة من قطعة واحدة " تحت الخط المستقيم من العلاقة بين P و Δ " .

2. إهمال أي فواقد خارجية من الطاقة .

3. إهمال وزن الكتلة المحسوسة .

4. تأثير المنشآت بالنسبة للأعمال المساقطة صغير جداً .

* In Axial load:

$$(i) \quad f = \frac{P}{A} \rightarrow P = f \cdot A$$

$$(ii) \quad \Delta = \frac{PL}{EA} \rightarrow \Delta = \frac{f \cdot L}{E}$$

- بالتعويض من المعادلة العامة : $\left(W(h + \Delta) = \frac{1}{2} P \Delta \right)$

$$(iii) \quad W \left[h + \frac{f_d \cdot L}{E} \right] = \frac{1}{2} [f_d \cdot A] \left[\frac{f_d \cdot L}{E} \right]$$

$$(iv) \quad \frac{W}{A} \left[h + \frac{f_d \cdot L}{E} \right] = \frac{1}{2} \frac{L}{E} f_d^2$$

an : $f_s = \frac{W}{A}$ بفرض الحمل "W" حمل إستاتيكي

و جـب المعادلة التربيعية :

$$(v) \quad f_d = f_s \cdot \left[1 + \sqrt{1 + \frac{2h}{\Delta_s}} \right]$$

f_d = Stress of Impact للإجهاد الناشئ من الصدم

$f_s = \frac{W}{A}$ للإجهاد بفرض الحمل "W" حمل إستاتيكي

$\Delta_s =$ للإزاحة بفرض الحمل "W" حمل إستاتيكي
 $= \frac{W \cdot L}{EA}$

$$\Delta_d = \Delta_s \cdot \left[1 + \sqrt{1 + \frac{2h}{\Delta_s}} \right]$$

Δ_d : الإزاحة الناتجة من الصدم

Δ_s : الإزاحة لو كانت الحمل إستاتيكي

$$\Delta_s = \frac{wL}{E \cdot A}$$

" K_d "

يسمى الترم بعامل الصدم $\left(1 + \sqrt{1 + \frac{2h}{\Delta_s}} \right)$ - لاحظ

$$K_d = \frac{F_d}{F_s} =$$

- حالات خاصة :

If : $h \ll \Delta_s$

(إرتفاع سقوط صغير)

الإجهاد الإستاتيكي $F = 2 F_s$ إجهاد الصدم

الإزاحة الإستاتيكي $\Delta = 2 \Delta_s$ إزاحة الصدم

If : $h \gg \Delta_s$

$$F = F_s \cdot \left[\sqrt{\frac{2h}{\Delta_s}} \right]$$

$$\Delta = \sqrt{2h \Delta_s}$$

Look

فعلك انك بالكلية ص ١١٩

- طاقة الصدم أثناء الحركة :

- الطاقة السابقة نتيجة سقوط جسم على الجسم "تأثير زنه".

$$\text{Energy } U = W(h + \Delta)$$

- ضا حالة سير جسم بسرعة معينة وصوت اصطدام :

- طاقة الحركة أثناء الصدم :

$$\text{Energy } = U = \frac{1}{2} m v^2$$

$$m : \text{mass} = \frac{W}{g} \quad \begin{array}{l} \text{الوزن} \\ \text{عجلة الجاذبية} \end{array}$$

السرعة : v

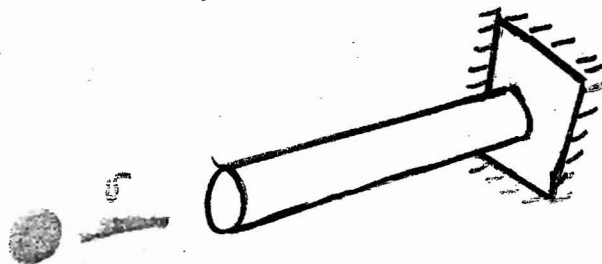
$$v = \sqrt{2gh} \quad \text{السرعة عند الصدم}$$

g : عجلة الجاذبية الأرضية

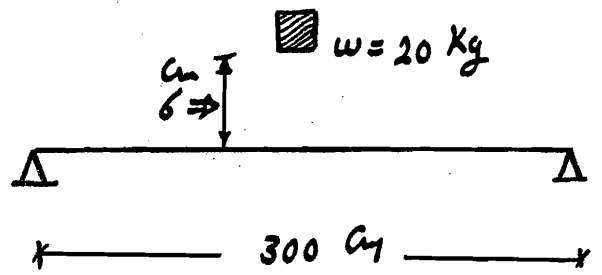
h : المسافة بين الجسم والكتلة

جسم يتحرك بسرعة

"U"

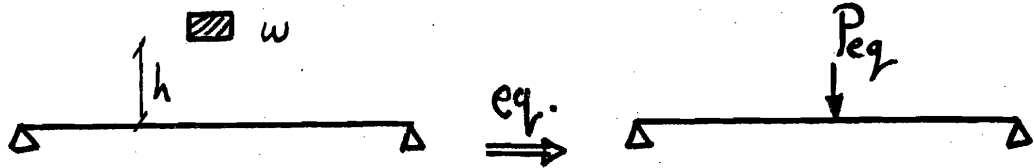


Ex (1): Find The max
Stress equivalente



By APPLING (E.S.L) method. (Given $E = 2000 \text{ t/cm}^2$)

Sol

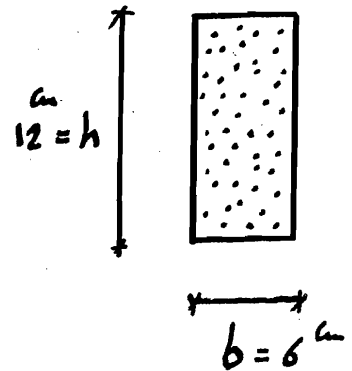


$$\Delta = \frac{P_{eq} L^3}{48 EI}$$

$$= \frac{P_{eq} \cdot (300)^3}{48 \cdot 2000 \cdot 1000 \cdot 864}$$

↑
للتحويل لـ kg

$$\therefore \Delta = \frac{P_{eq}}{3072} \rightarrow \textcircled{1}$$



$$I = \frac{b h^3}{12} = \frac{6 (12)^3}{12}$$

$$I = 864 \text{ cm}^4$$

From General eqn. :

$$w (h + \Delta) = \frac{1}{2} \cdot P_{eq} \cdot \Delta$$

$$\therefore 20 \left(6 + \frac{P_{eq}}{3072} \right) = \frac{1}{2} \cdot P_{eq} \cdot \frac{P_{eq}}{3072}$$

$$\therefore \frac{P_{eq}^2}{3072} - 120 = 0 \Rightarrow$$

Look

معادلة

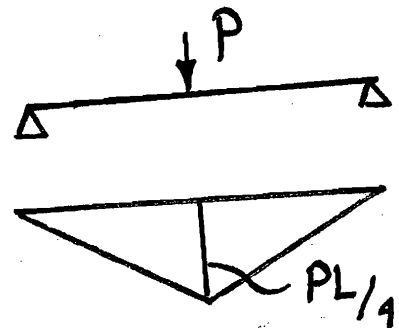
∴ Solve this eqn. : Kg

$$\therefore P_{eq} = 878.9$$

$$f_{(stress)} = \frac{M}{I} \cdot y$$

$$= \frac{P \cdot L / 4}{I} \cdot y$$

$$= \frac{(878.9 \cdot 300) / 4}{864} \cdot \left(\frac{12}{2}\right) = \underline{\underline{457.76 \text{ Kg/cm}^2}}$$



→ Find Dynamic Factor (K_d): "مکمل (ضرب)"

$$K_d = \frac{P_d}{P_s} = \frac{P_d}{P_s = W} = \frac{878.9}{20} = 43.95$$

2 is de "K_d" مکمل (ضرب) 2

K_d × 2

A weight $W = 400 \text{ kg}$ falls from a height $h = 90 \text{ cm}$ upon a vertical wooden pole 6 m long and 30 cm in diameter and fixed at the lower end. Determine the maximum compressive stress in the pole, assuming that $E_{\text{wood}} = 100 \text{ t/cm}^2$.

* Given :

$$W = 400 \text{ kg} \quad \& \quad h = 90 \text{ cm}$$

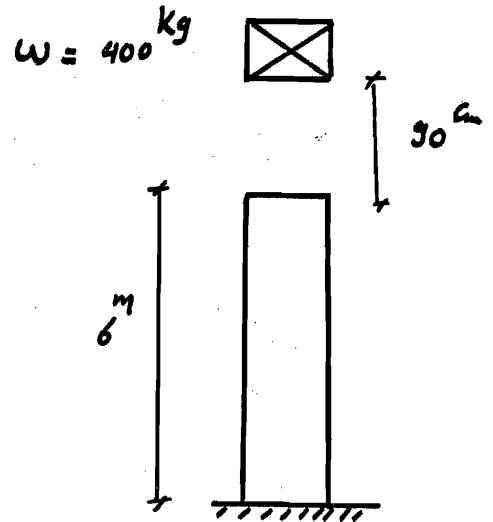
$$L = 6 \text{ m} \quad \& \quad D = 30 \text{ cm} \quad \& \quad E = 100 \text{ t/cm}^2$$

Sol

$$A = \frac{\pi}{4} (D)^2 \quad \text{Axial}$$

$$= \frac{\pi}{4} (30)^2 = 706.9 \text{ cm}^2$$

$$\Delta = \frac{P \cdot L}{E \cdot A} = \frac{6 \cdot 100 \cdot P_{eq}}{100 \cdot 706.9} = 8.488 \cdot 10^{-3} P_{eq}$$



$$W(h + \Delta) = \frac{1}{2} P_{eq} \Delta$$

$$\frac{400}{1000} \cdot (90 + 8.488 \cdot 10^{-3} P_{eq}) = \frac{1}{2} P_{eq} \cdot (8.488 \cdot 10^{-3} P_{eq})$$

$$P_{eq}^2 - 0.8 P_{eq} - 8482.56 = 0.0$$

$$P_{eq} = 92.5 \text{ ton}$$

$$f_{max} = \frac{P_{eq}}{A} = \frac{92.5}{706.9} = 0.13 \text{ t/cm}^2$$

A weight $W = 200 \text{ kg}$ is dropped from a height H on the Steel bar with a circular cross section (Diameter = 5 cm) and length = 100 cm with a modulus of Elasticity = 2000 t/cm^2 . Find the maximum height (h) so that the total deformation in the bar doesn't exceed 0.10 cm .

* Given :

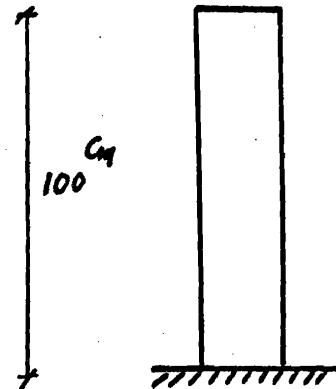
$$D = 5 \text{ cm}$$

$$W = 200 \text{ kg}$$

$$E = 2000 \text{ t/cm}^2 \quad \Delta = 0.1 \text{ cm}$$



H



$$\text{Area} = A = 19.65 \text{ cm}^2$$

Sol

$$\Delta = \frac{P_{eq} \cdot L}{E \cdot A}$$

$$\therefore 0.1 = \frac{P_{eq} \cdot 100}{2000 \cdot 19.65}$$

$$\therefore P_{eq} = 39.3 \text{ ton}$$

$$\therefore W(h + \Delta) = \frac{1}{2} P_{eq} \Delta$$

$$\therefore 200/1000 [H + 0.1] = \frac{1}{2} \cdot [39.3] \cdot 0.1$$

$$\therefore H = 9.72 \text{ cm}$$

٩.٧٢ م
 تيمر سقوط المله ٢٠٠ كغ من على ارتفاع
 ليحقق انضغاط ٠.١ م.

Design a simple beam with a rectangular cross section to sustain the impact of a failing weight from a height of 50 cm on the mid-span of the beam. Use the following information:

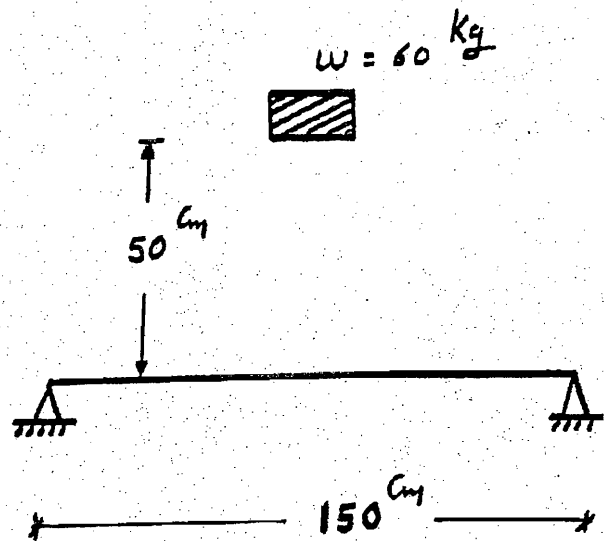
- The beam height = 30 cm
- The span of the beam = 150 cm.
- The dropped weight = 600 kg.
- The flexural strength of the beam material = 2000 kg/cm²
- Design factor of safety = 2.
- Material modulus of elasticity = 2200 t/cm²

Given : $H = 50 \text{ cm}$

$w = 60 \text{ kg}$ & $F.O.S = 2$

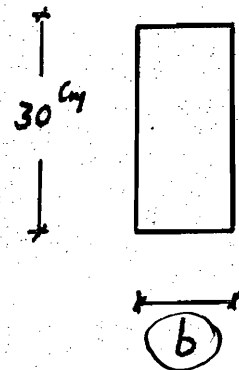
$E = 2200 \text{ t/cm}^2$

$f_u = 2000 \text{ kg/cm}^2$



Sol

$$f_{all} = \frac{f_u}{F.O.S} = \frac{2000}{2} = 1000 \text{ kg/cm}^2$$



$$\begin{aligned} \therefore 1000 &= \frac{M}{I} \cdot y \\ &= \frac{P \cdot L}{4 \cdot I} \cdot \left(\frac{30}{2}\right) \\ &= \frac{P}{I} \cdot \frac{150}{4} \cdot \left(\frac{30}{2}\right) \end{aligned}$$

$$\therefore \frac{P}{I} = 1.78 \rightarrow \textcircled{1}$$

$$\therefore \Delta = \frac{P \cdot L^3}{48 E I}$$

$$= \frac{P}{48 I} \cdot \frac{(150)^3}{2200 \cdot 1000} = \textcircled{1.78} \cdot \frac{(150)^3}{2200 \cdot 1000 \cdot 48}$$

$$\therefore \Delta = 0.057 \text{ cm}$$

From General eqn. :-

$$w \cdot (h + \Delta) = \frac{1}{2} \cdot P \cdot \Delta$$

$$\therefore 600 \cdot (50 + 0.057) = \frac{1}{2} \cdot P \cdot 0.057$$

$$P = 1053800 \text{ kg}$$

$$\therefore \frac{P}{I} = 1.78 \Rightarrow I = \frac{1053.8}{1.78} = 594 \text{ cm}^4$$

$$\therefore I = \frac{b h^3}{12} \Rightarrow 594 = \frac{b \cdot (30)^3}{12}$$

$$\therefore b = 26.3 \text{ cm}$$

Final 2007

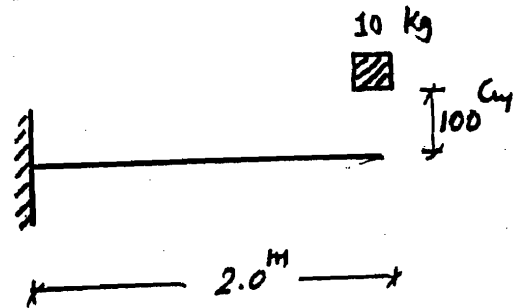
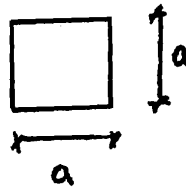
- Q- Calculate the minimum dimensions of a square cross section of 2.0 meter long cantilever made of steel grade 40/60 knowing that the steel modulus of elasticity (E) is 2000 t/cm², and an impact load W = 10 kg that dropped from a height 100 cm on the free end of the cantilever to cause the yield stress. Calculate also, the equivalent static load and the corresponding deflection. (F.O.S = 2)

* Given :

Grade (40/60)

$$E = 2000 \text{ t/cm}^2$$

$$W = 10 \text{ Kg}$$



Sol

∴

$$W(h + \Delta) = \frac{1}{2} P_{eq} \Delta$$

$$\therefore \Delta = \frac{P_{eq} \cdot L^3}{3EI} = \frac{P_{eq}}{I} \cdot \frac{(200)^3}{3 \times 2000} = 1333.33 \cdot \frac{P_{eq}}{I} \rightarrow \textcircled{1}$$

$$\therefore F_y = 40 \text{ kg/mm}^2 \rightarrow f_{all} = f_y / F.O.S = 40/2 = 20 \text{ kg/cm}^2$$

$$\therefore \Rightarrow F_y = \frac{20 \cdot (10)}{1000} = 2 \text{ t/cm}^2$$

$$\therefore F_y = \frac{M}{I} \cdot Y = \frac{P_{eq} \cdot 200}{I} \cdot \left(\frac{a}{2}\right)$$

$$\therefore 2 = \frac{200 P_{eq}}{I} \cdot \left(\frac{a}{2}\right) \Rightarrow \frac{P_{eq}}{I} = (0.02/a)$$

$$\therefore P_{eq} = \frac{0.02}{2} + \frac{a \cdot a^3}{12} = \frac{a^3}{600} \rightarrow (2)$$

From ①

$$\therefore \Delta = 1333.33 \times \frac{0.02}{a} = \frac{26.6}{a} \rightarrow (3)$$

$$\omega(h+\Delta) = \frac{1}{2} \cdot P_{eq} \cdot \Delta$$

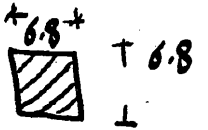
$$\% \frac{10}{1000} \left(100 + \frac{26.6}{a} \right) = \frac{1}{2} * \frac{a^3}{800} * \frac{26.6}{a}$$
 له القوي لا "ton"

$$1 + \frac{0.266}{a} = a^2 \cdot 0.02217$$
 بقول الكارلة بالآلة :

$a_{LO} = \frac{0.266}{2} = 0.133$

∴ Cross section

$$(5.8 \times 5.8^{\text{cm}})$$



$$\therefore P_{eq} = \frac{(8.8)^3}{800} = 0.524 \text{ ton} \quad \underline{\underline{\quad \quad \quad}}$$

$$\therefore \Delta = \frac{26.0}{6.8} = \underline{\underline{3.9 \text{ cm}}}$$

$$D = \frac{PL}{AE}$$

$$D = \frac{P L^3}{48 EI}$$

$$\sqrt{f} = \frac{M}{L} \text{ g}$$

$$u \in (h \cap D) \in U$$

(14) $\mathcal{U} = \frac{1}{2} P D = u(h + D)$

~ Sheet 2 ~

- a. A truck of weight 10 tons is traveling at a constant speed of 12 km/hr with engine stopped and the rear of the truck is connected to one end of a wire of diameter 3 cm and the other end is connected to a pulley. Suddenly, the wire gets jammed at the pulley and the truck stopped. Calculate the stress in the wire and its elongation due to wire jamming. Given that the elastic modulus of the wire material is 1500 t/cm², wire length = 100 m, g = 980 cm/sec².

Given: $W = 10 \text{ ton}$

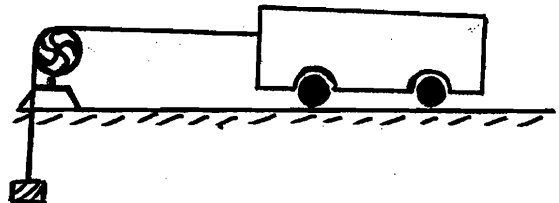
$$V = 12 \text{ km/hr}$$

$$\therefore V = \frac{12 \times 10^5}{60 \times 60} = 333.33 \text{ cm/sec}$$

$$D = 3 \text{ cm} \quad E = 1500 \text{ t/cm}^2$$

$$L = 100 \text{ m} \quad g = 980 \text{ cm/sec}^2$$

100 m



$$m = \frac{W}{g}$$

Sol

$$\therefore U = \frac{1}{2} m V^2$$

$$= \frac{1}{2} \cdot \frac{10 \times 1000 \text{ } ^{\text{Kg}}}{980} \cdot (333.33)^2$$

$$= 566893.4 \text{ Kg.cm}$$

$$\therefore U = \frac{1}{2} P \cdot \Delta = \frac{1}{2} P^2 \cdot \frac{L}{E \cdot A} = \frac{1}{2} f^2 \cdot \frac{L A}{E}$$

$$\therefore 566893.4 = \frac{1}{2} \cdot f^2 \cdot \frac{(100 \times 100 \text{ } ^{\text{cm}}) \times \frac{\pi}{4} (3)^2}{1500 \times 1000 \text{ } ^{\text{Kg}}}$$

$$\therefore f = 4905 \text{ Kg/cm}^2 \rightarrow \text{find } \Delta = \text{---}$$

طاقة حركية

If the allowable stress for the wire material is 1200 kg/cm², what do you suggest to handle this problem of operation?

$$f_{all} = 1200 \text{ kg/cm}^2$$

- لإيجاد المسموح به في الحبل أقل منه الفعلي $f = 4905 \text{ kg/cm}^2$

- لثلاث إنشيار الحبل :

① تقليل السرعة "v".

② زيادة مقطع الـ "Wire".

③ تقليل وزن الـ "Truck".

كقوة

كل حاسب منه ١٢

معادلة العامة

- معادلة العامة :

$$U = \frac{1}{2} P \cdot \Delta = \frac{1}{2} f^2 \cdot \frac{LA}{E} = \frac{1}{2} m v^2$$

- الحلول الباقية من المعادلة :

① $f^2 \propto v^2$

② $f^2 \propto 1/A$

③ $f^2 \propto W$

"تناسب طردي"

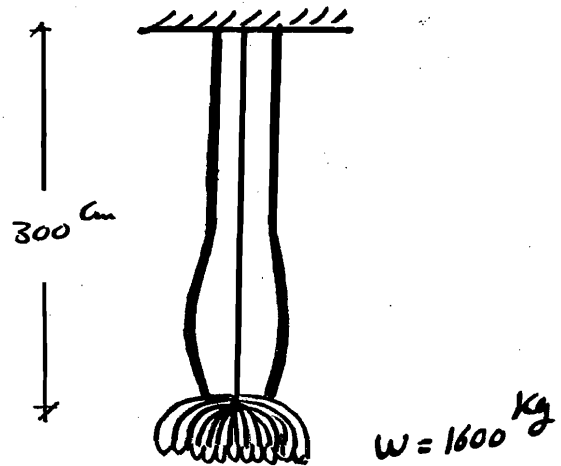
- 800
- b. A huge chandelier (نجفة كبيرة) of 1600 kg weight is hanged to the ceiling using an old wire of current length (while hanging the chandelier) of 300 cm. There is doubt (شك) about the safety of the wire carrying the chandelier; however, it is extremely difficult to change the wire due to the relatively heavy weight of the chandelier. Thus it is decided to secure the system through putting two wires parallel to the existing one. Due to the difficulty in securing the new two wires with the same length as the old one, it is found that these wires should be of 304 cm in length. Calculate the required diameter of the new wires and the expected elongation at the time of failure of the old wire if the allowable stress of the wire material is 1200 kg/cm² and its elastic modulus is 2000 t/cm².

Given

$$P_{all} = 1200 \text{ kg/cm}^2$$

$$W = 1600 \text{ kg}$$

$$E = 2000 \text{ t/cm}^2$$



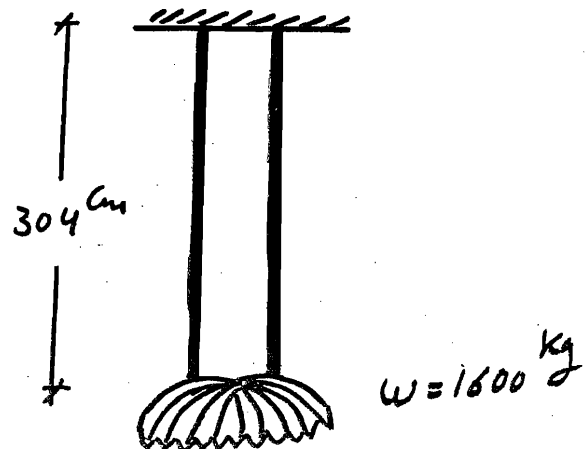
- Find :

Diameter for new wire.

النجفة

النجفة بعد نزاج
الحميل لك wire في

Sol



$$\therefore f_{all} = \frac{P}{A} \Rightarrow 1200 A = P$$

$$\therefore P = 1200 A \rightarrow \textcircled{1}$$

$$\therefore \Delta = \frac{P L}{E A} \Rightarrow = \frac{P \cdot (304)}{2000 \cdot 1000 \cdot A}$$

طول هذا الجسد
له للتحويل "kg"

From $\textcircled{1}$ $P = 1200 A$

$$\therefore \Delta = \frac{1200 \cdot 304 A}{2000 \cdot 1000 \cdot A} = \underline{0.1824 \text{ cm}}$$

معادلة الطاقة :

$$\therefore W(h + \Delta) = \frac{1}{2} P \Delta$$

$$1600 (4 + 0.1824) = \frac{1}{2} \cdot P \cdot (0.1824)$$

$$\therefore P = 73375.4 \text{ kg}$$

From $\textcircled{1} \Rightarrow A = 61.14 \text{ cm}^2$

$$\therefore A_{\text{wire}} = \frac{A}{2} = 30.57 \text{ cm}^2$$

$$\therefore A_{\text{wire}} = \frac{\pi}{4} (D)^2$$

$$\Rightarrow$$

$$D = 6.23 \approx 6.25$$

المساحة
للـ "2 wire"

Determine the number of persons can be carried by an elevator which is lifted by one steel cable of 16 mm. diameter and 30 m long. Knowing that the cable is made of steel of grade (28/40) and it has an elastic modulus $E = 2000 \text{ t/cm}^2$. Consider the elevator's dead load is 720 kg, the average person weight = 85 kg and the design factor of safety = 2.5.

* Given :

$$D = 16 \text{ mm}$$

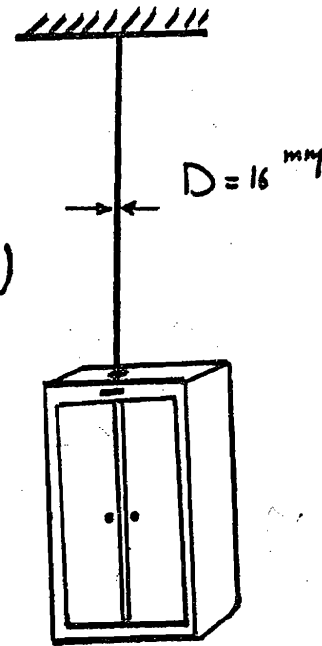
$$L = 30 \text{ m}$$

$$E = 2000 \text{ t/cm}^2 \text{ \& Grade (28/40)}$$

$$\text{Dead Load} = 720 \text{ kg}$$

$$\text{Person wt.} = 85 \text{ kg}$$

$$F.O.S = 2.5$$



Sol

$$F_y = 28 \text{ Kg/mm}^2 \Rightarrow F_{all} = \frac{F_y}{F.O.S} = \frac{28}{2.5} = 11.2$$

$$\therefore F_{all} = \frac{P_{eq}}{A} \rightarrow P_{eq} = F_{all} \cdot A_{area}$$

$$\therefore P_{eq} = 11.2 \cdot \frac{\pi}{4} (16)^2 = 2251.9 \text{ Kg}$$

$$\Delta = \frac{P_{eq} \cdot L}{E \cdot A} = \frac{2251.9 \cdot 30 \cdot 100}{2000 \cdot 1000 \cdot \frac{\pi}{4} \left(\frac{16}{10}\right)^2}$$

المحول الى سم

في المقول Kg

$$\therefore \Delta = 1.68 \text{ سم}$$

→ From general equ. :

$$W(h + \Delta) = \frac{1}{2} P \Delta$$

$$\therefore W * (0.0 + 1.68) = \frac{1}{2} * 2251.9 * 1.68$$

$$\therefore W = 1125.95 \text{ Kg}$$

- حساب عدد الأشخاص "n"

$$\therefore W = 720 + n * 85$$

$$\therefore 1125.95 = 720 + n * 85$$

$$\therefore n = \underline{4.78} \text{ Person !!}$$

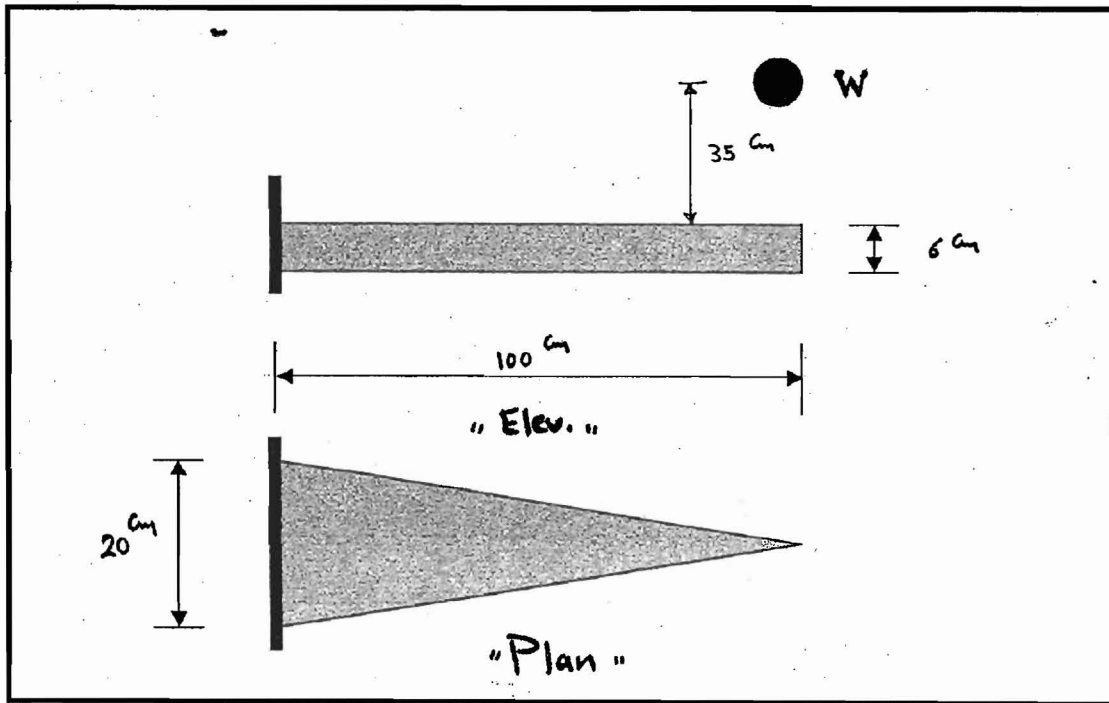
$$\therefore \underline{n = 4 \text{ Person}}$$

از بچه و خسته عیب
خدا!!

Look

" H. W. "

a. Calculate the maximum weight W that can be dropped from a height of 35 cm on a cantilever beam in the configuration shown next to cause a allowable stress 1200 kg/cm^2 knowing that the modulus of elasticity for the material is 2000 t/cm^2 and a cross section of beam is available.



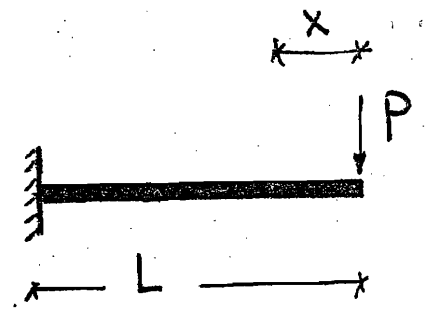
(1)

- d) A three- point bending test is performed on a metallic beam that is made of steel and has a span of 150 cm and a rectangular cross section of width of 6.0 cm and depth of 9.0 cm. The load is applied at mid span of the beam. The mid span deflection is 0.20 cm of 2.0 ton concentrated load. This beam is subjected to impact bending due to dropped weight (w) of 10.0 kg from a height of 80.0 cm in the middle of span. Determine the maximum stress due to the impact load and also the absorbed energy of beam providing that the stress due to the two cases is less than the proportional limit stress of material beam.

(2)

(2)

(1) Prove that : $\Delta = \frac{PL^3}{3EI}$
for shown beam



Sol

$$M_x = P \cdot x \quad \rightarrow (0 \rightarrow L)$$

$$\therefore U = \int_0^L \frac{(M_x)^2}{2EI} \cdot dx$$

$$= \int_0^L \frac{P^2 x^2}{2EI} \cdot dx = \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^L$$

$$\therefore U = \frac{P^2 \cdot L^3}{6EI}$$

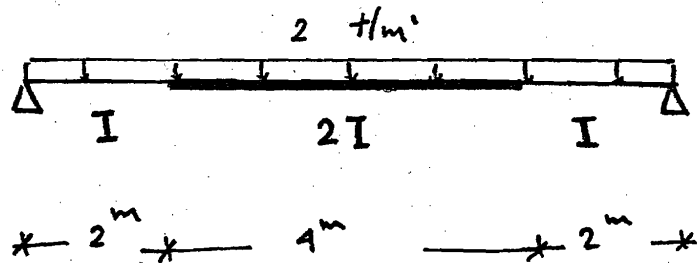
Check

$$\therefore U = \frac{1}{2} P \Delta \Rightarrow \frac{1}{2} P \Delta = \frac{P^2 L^3}{6EI}$$

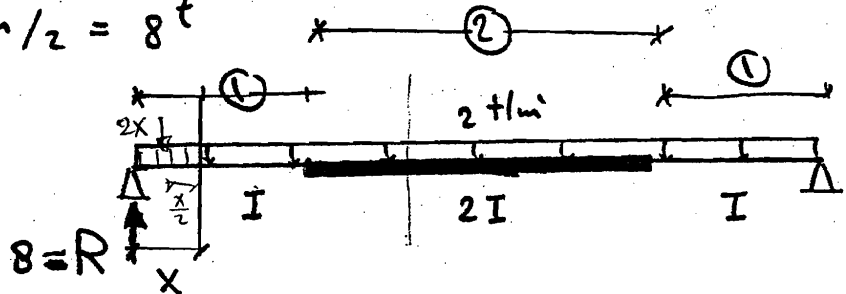
$$\therefore \Delta = \frac{PL^3}{3EI}$$

(2) Calculate Elastic Energy in this beam

Sol



$$R = 2 \times 8 = 16 \text{ ton} / 2 = 8 \text{ t}$$



* Part ①

$$\hat{\circ} \hat{\circ} \hat{\circ} M_x = 8 \cdot x - 2 \cdot x^2 / 2 = 8x - x^2$$

(0 \rightarrow 2)

* Part ②

$$\hat{\circ} \hat{\circ} \hat{\circ} M_x = 8x - 2 \cdot \frac{x^2}{2} = 8x - x^2$$

(2 \rightarrow 6)

$$\therefore U = 2U_1 + U_2$$

$$\hat{\circ} \hat{\circ} \hat{\circ} U_1 = \int_0^2 \frac{(8x - x^2)^2}{2EI} \cdot dx$$

$$\therefore U_1 = \int_0^2 \frac{64x^2 + x^4 - 16x^3}{2EI} \cdot dx$$

$$= \frac{1}{2EI} \left[64 \cdot \frac{x^3}{3} + \frac{x^5}{5} - 16 \cdot \frac{x^4}{4} \right]_0^2$$

$$\therefore U_1 = 56.533 / EI \rightarrow \textcircled{1}$$

$$\therefore U_2 = \int_2^6 \frac{(M_x)^2}{2EI} \cdot dx$$

$$= \frac{1}{2EI(2I)} \left[64 \frac{x^3}{3} + \frac{x^5}{5} - 16 \cdot \frac{x^4}{4} \right]_2^6$$

$$\therefore U_2 = 216.533 / EI \rightarrow \textcircled{2}$$

from ①, ②

$$\therefore U = 2U_1 + U_2$$

$$= 2 \cdot \frac{56.533}{EI} + \frac{216.533}{EI}$$

$$\therefore U = 329.6 \text{ t.m}$$

نصف
2=x

نصف x
دائرة ①
نصف ②