



SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2013

**TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION**

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 180 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- Show all necessary working in Questions 11–16

Total Marks - 100 Marks

Section I 10 Marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II 90 Marks

- Attempt Questions 11–16
- Allow about 2 hour 45 minutes for this section.

Examiner: *External Examiner*

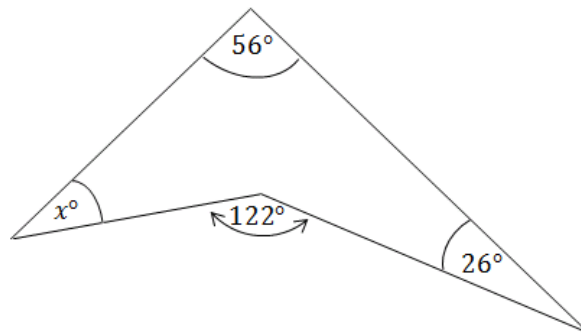
This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Section I**Objective-response Questions****Total marks – 10****Attempt Questions 1 – 10**Answer each question on the multiple choice answer sheet provided.

1) $\frac{\tan^2 \theta}{1 + \tan^2 \theta} + \cos^2 \theta$ equals

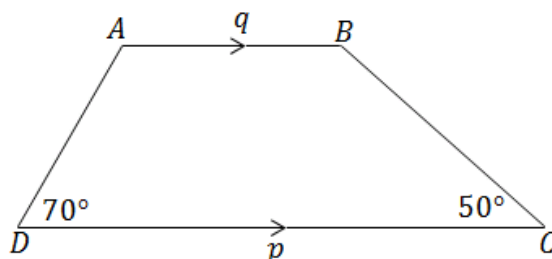
- (A) 1
- (B) $\frac{1}{2} + \cos^2 \theta$
- (C) $1 + \tan^2 \theta$
- (D) $1 + \cos^2 \theta$

2)

In the figure above, x equals

- (A) 31°
- (B) 34°
- (C) 40°
- (D) 48°

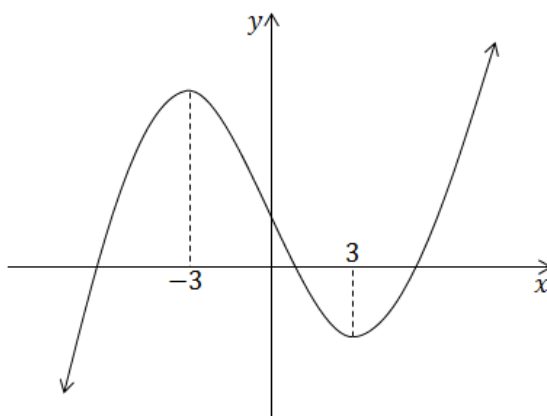
3)



In the figure above $AB \parallel DC$, $AB = q$ and $DC = p$. BC equals

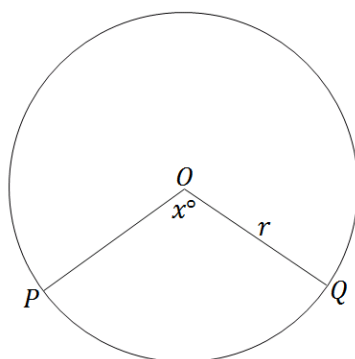
- (A) $\frac{(p+q) \sin 50^\circ}{2 \sin 70^\circ}$
 - (B) $\frac{(p+q) \sin 70^\circ}{2 \sin 50^\circ}$
 - (C) $\frac{(p-q) \sin 70^\circ}{\sin 60^\circ}$
 - (D) $\frac{(p-q) \sin 50^\circ}{\sin 70^\circ}$
- 4) The period of the function $f(x) = \sin\left(3x - \frac{\pi}{3}\right)$, $x \in \mathbb{R}$ is
- (A) $\frac{\pi}{9}$
 - (B) $\frac{2\pi}{3}$
 - (C) 2π
 - (D) $\frac{\pi}{3}$
- 5) The solution(s) of the equation $e^x + e^{-x} = -\frac{3}{2}$, where $x \in \mathbb{R}$, is (are)
- (A) $\ln 2$ only
 - (B) $\pm \ln 2$
 - (C) $-\ln 2$ only
 - (D) None of these

- 6) From the graph of $y = f(x)$, when is $f'(x)$ negative?



- (A) $x < -3$ or $x > 3$
- (B) $-3 < x < 3$
- (C) $x \leq -3$ or $x \geq 3$
- (D) $-3 \leq x \leq 3$
- 7) If M is decreasing at an increasing rate, what does this suggest about $\frac{dM}{dt}$ and $\frac{d^2M}{dt^2}$?
- (A) $\frac{dM}{dt} < 0$ and $\frac{d^2M}{dt^2} < 0$
- (B) $\frac{dM}{dt} > 0$ and $\frac{d^2M}{dt^2} < 0$
- (C) $\frac{dM}{dt} < 0$ and $\frac{d^2M}{dt^2} > 0$
- (D) $\frac{dM}{dt} > 0$ and $\frac{d^2M}{dt^2} > 0$

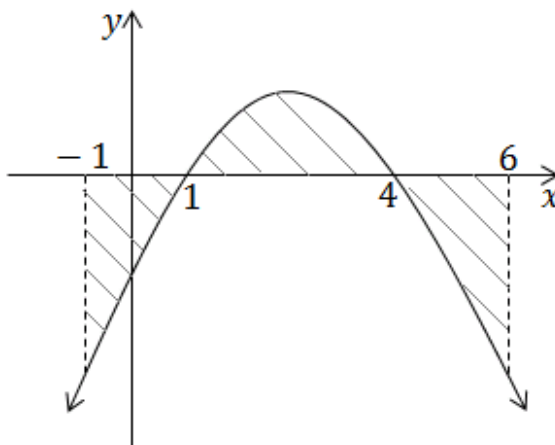
8)



In the figure, the radius of the sector is r and $\angle POQ = x^\circ$. If the area of the sector is A then x equals

- (A) $\frac{2A}{r^2}$
- (B) $\frac{360A}{\pi r^2}$
- (C) $\frac{180A}{\pi r^2}$
- (D) $\frac{180A}{r^2}$

9) Which of the following expressions gives the total area of the shaded region in the diagram?



- (A) $\int_{-1}^6 f(x) dx$
- (B) $-\int_{-1}^0 f(x) dx + \int_0^6 f(x) dx$
- (C) $-\int_{-1}^1 f(x) dx + \int_1^4 f(x) dx - \int_4^6 f(x) dx$
- (D) $\int_1^4 f(x) dx + 2 \int_4^6 f(x) dx$

10) Which of the following is the derivative of $y = \ln[f(x)]$

(A) $\frac{f(x)}{f'(x)}$

(B) $\frac{f'(x)}{f(x)}$

(C) $\frac{1}{f'(x)}$

(D) $\frac{f''(x)}{f'(x)}$

End of Section I

Section II**Free response questions****Total marks – 90****Attempt Questions 11 – 16**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Factorise $x^2 - 2x + 1 - 4y^2$ 1

(b) Simplify
$$\sqrt{\frac{3^{5k+2}}{27^k}}$$
 1

(c) Simplify
$$\frac{\log(a^3b^2) - \log(ab^2)}{\log \sqrt{a}}$$
 1

(d) Solve $x^2 + 2x - 8 > 0$ 1

(e) By considering the cases $x \leq 1$ and $x > 1$, or otherwise, solve $|1 - x| = x - 1$ 2

(f) For the parabola $(x - 3)^2 = -4y$.

(i) Find the coordinates of the vertex. 1

(ii) State the equation of the directrix of the parabola. 1

(g) Prove
$$\frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$$
 2

(h) Evaluate
$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 + x - 6}$$
 1

(i) Find the equation of a straight line passing through the point of intersection of the lines $l_1: 2x - y - 4 = 0$ and $l_2: 2x + 3y - 12 = 0$ and perpendicular to the line $2x - 3y + 1 = 0$. 2

(j) Find the equation of the tangent to the curve $y = 2 \sin 2x$ at the point $\left(\frac{\pi}{8}, \sqrt{2}\right)$. 2

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

- (a) Draw a number plane and mark the points $P(-2, 2)$ and $Q(-4, -2)$.
- (i) Show that the equation of the line through P perpendicular to PQ is given by $x + 2y - 2 = 0$ 2
- (ii) The line perpendicular to PQ through P intersects the x -axis at R . Find the coordinates of R . 2
- (iii) Show that the mid-point of QR is $(-1, -1)$. Mark this point T on your diagram. 1
- (iv) Find the perpendicular distance from T to the interval PR . 2
- (b) x and y are positive numbers. $x, -2, y$ are consecutive terms of a geometric series, and $-2, y, x$ are consecutive terms of an arithmetic series.
- (i) Find the value of xy . 1
- (ii) Find the values of x and y . 3
- (iii) Find the sum to infinity of the geometric series $x - 2 + y \dots$ 2
- (c) Given that α and $m\alpha$ are the roots of the equation $x^2 + px + q = 0$, show that $mp^2 = (m + 1)^2q$ 2

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x :

(i) $x \sin x$ 1

(ii) $\ln(x^2 + 4)$ 1

(iii) $e^{5x} + x$ 1

(b) The graph of $y = f(x)$ passes through the point $(3, 5)$, and $f'(x) = 2x - 3$. Find $f(x)$. 2

(c) Find:

(i)
$$\int \sqrt{x + 10} \cdot dx$$
 1

(ii)
$$\int_0^{\frac{\pi}{8}} \sec^2 2x \cdot dx$$
 1

(d) Consider the curve $y = \cos 2x$:

(i) Sketch $y = \cos 2x$ for $0 \leq x \leq 2\pi$. 1

(ii) Find the area between the curve $y = \cos 2x$ and the x -axis from $x = 0$ to $x = \pi$. 2

(e) The population P of Newcastle after t years is given by the exponential equation

$$P = 50000e^{-0.08t}$$

(i) Find the time to the nearest year for the initial population to halve. 1

(ii) Find the number of people who leave Newcastle during the tenth year. 2

(f) A continuous curve $y = f(x)$ has the following properties for the closed interval $-3 \leq x \leq 5$: $f(x) > 0$, $f'(x) > 0$, $f''(x) < 0$. Sketch a curve satisfying these conditions. 2

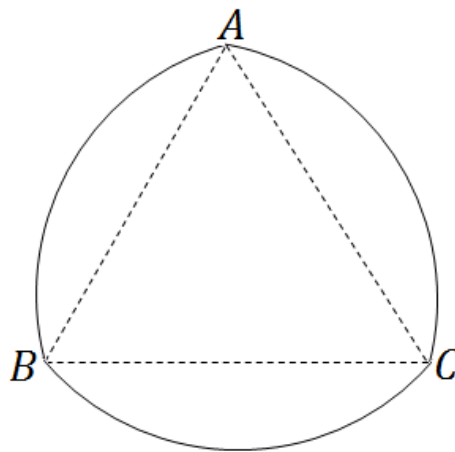
End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

- (a) Given that $f(x) = x(x - 2)^2$
- (i) Show that $f'(x) = 3x^2 - 8x + 4$. 1
 - (ii) Find 2 values of x for which $f'(x) = 0$, and give the corresponding values of $f(x)$. 1
 - (iii) Determine the nature of the turning points of the curve $y = f(x)$. 2
 - (iv) Sketch the curve $y = f(x)$ showing all essential features. 2
 - (v) Use your sketch to solve the inequation $x(x - 2)^2 \geq 0$. 1
- (b) An economist predicts that over the next few months, the price of crude oil, p dollars a barrel, in t weeks time will be given by the formula

$$P = 0.005t^3 - 0.3t^2 + 4.5t + 98$$

- (i) What is the price at present, and how rapidly is it going up? 2
 - (ii) How high does she expect the price to rise? 2
- (c) A coin is made by starting with an equilateral triangle ABC of side 2 cm. With centre A an arc of a circle is drawn joining B to C. Similar arcs join C to A and A to B.

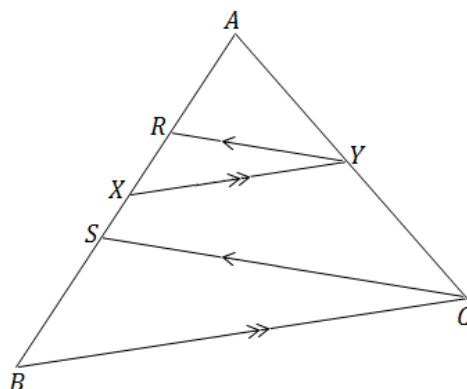


- (i) Find, exactly, the perimeter of the coin. 2
- (ii) Find area of one of its faces. 2

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

- (a) Given that in $\triangle ABC$, $XY \parallel BC$ and $RY \parallel SC$,



Prove $AX:XB = AR:RS$.

2

- (b) A couple plan to buy a home and they wish to save a deposit of \$40 000 over five years. They agree to invest a fixed amount of money at the beginning of each month during this time. Interest is at 12% per annum compounded monthly.

- (i) Let \$P be the monthly investment. Show that the total investment \$A after five years is given by

2

$$A = P(1.01 + 1.01^2 + \dots + 1.01^{60})$$

- (ii) Find the amount \$P needed to be deposited each month to reach their goal. Answer correct to the nearest dollar.

2

- (c) A train is travelling on a straight track at 48 ms^{-1} . When the driver sees an amber light ahead, he applies the brakes for a period of 30 seconds, producing a deceleration of $\frac{1}{125}t(30 - t) \text{ ms}^{-2}$, where t is the time in seconds after the brakes are applied.

- (i) Find how fast the train is moving after 30 seconds.

2

- (ii) How far it has travelled in that time.

2

- (d) Two ordinary dice are thrown. Find the probability that the sum of the numbers on the uppermost faces is at least 10.

2

- (e) Consider the function $f(x) = x^2 \ln x - \frac{x^2}{2}$:

- (i) Show that $f'(x) = 2x \ln x$.

1

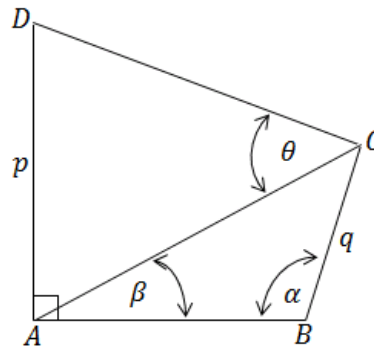
- (ii) Hence find $\int_1^2 x \ln x \, dx$.

2

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a)



$ABCD$ is a quadrilateral with AD perpendicular to AB . Given that $\angle CAB = \beta$, $\angle ABC = \alpha$, $\angle ACD = \theta$, $AD = p$ and $BC = q$.

- (i) Show that $\angle ADC = 90 - (\theta - \beta)$ 1
- (ii) Using the sine rule, prove that 3

$$q = \frac{p \sin \beta \cos(\theta - \beta)}{\sin \theta \sin \alpha}$$

(b) Consider the function $y = f(x) = 1 + e^{2x}$.

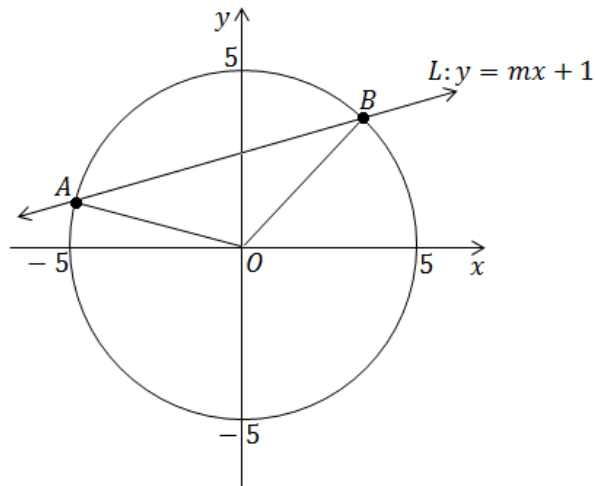
- (i) Find $f(0), f(1), f(2)$. 1
- (ii) Show that $x = \frac{1}{2} \ln(y - 1)$. 2
- (iii) The volume V formed when the area between $y = 1 + e^{2x}$, the y -axis, and the lines $y = 2$ and $y = 4$ is rotated about the y -axis is given by:

$$V = \frac{\pi}{4} \int_2^4 [\ln(y - 1)]^2 \cdot dy$$

Use Simpson's rule with 3 function values to estimate this volume. Leave your answer rounded to 3 significant figures. 3

Question 16 continues on the next page

(c)



In the above figure, the line $L: y = mx + 1$ cuts the circle $x^2 + y^2 = 25$ at two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

(i) Show that x_1 and x_2 are the roots of $(1 + m^2)x^2 + 2mx - 24 = 0$. 2

(ii) Show that area of $\triangle OAB = \frac{1}{2}(x_2 - x_1)$. 3

End of paper.

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x$, $x > 0$

2013 MATHEMATICS TRIAL - SOLUTIONS (2-UNIT)

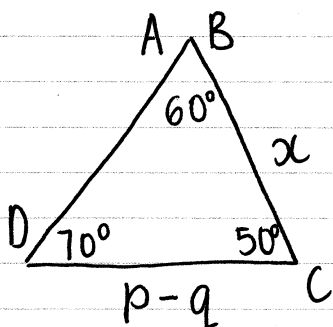
$$\begin{aligned}
 1. \quad & \frac{\tan^2 \theta}{1 + \tan^2 \theta} + \cos^2 \theta \\
 &= \frac{\tan^2 \theta}{\sec^2 \theta} + \cos^2 \theta \\
 &= \sin^2 \theta + \cos^2 \theta \\
 &= 1
 \end{aligned}$$

[A]

$$\begin{aligned}
 2. \quad & 122^\circ = x^\circ + 56^\circ + 26^\circ \\
 & 122^\circ = x^\circ + 82^\circ \\
 & x^\circ = 40^\circ
 \end{aligned}$$

[C]

3. Eliminate AB:



$$\begin{aligned}
 \frac{x}{\sin 70^\circ} &= \frac{p-q}{\sin 60^\circ} \\
 x &= \frac{(p-q) \sin 70^\circ}{\sin 60^\circ}
 \end{aligned}$$

[C]

$$\begin{aligned}
 4. \quad & f(x) = \sin \left(3x - \frac{\pi}{3} \right) \\
 & \text{Period} = \frac{2\pi}{3}
 \end{aligned}$$

[B]

$$5. \quad e^x + e^{-x} = -\frac{3}{2}$$

None of these

[D]

$$6. \quad f'(x) < 0$$

$$-3 < x < 3$$

[B]

$$\begin{aligned}
 7. \quad & \frac{dM}{dt} < 0 \text{ and } \frac{d^2M}{dt^2} > 0 \\
 \text{OR} \quad & \frac{dM}{dt} < 0 \text{ and } \frac{d^2M}{dt^2} < 0
 \end{aligned}$$

[C] [A]

$$\begin{aligned}
 8. \quad & A = \frac{x}{360} \times \pi \times r^2 \\
 & \frac{360A}{\pi r^2} = x
 \end{aligned}$$

[B]

$$\begin{aligned}
 9. \quad & A = \int_{-1}^1 f(x) dx + \int_1^4 f(x) dx \\
 & - \int_4^6 f(x) dx
 \end{aligned}$$

[C]

$$10. \quad y = \ln[f(x)]$$

$$y' = \frac{f'(x)}{f(x)}$$

[D]

11) (a) $x^2 - 2x + 1 - 4y^2$
 $(x-1)^2 - 4y^2 \Rightarrow A - B^2$ (15)

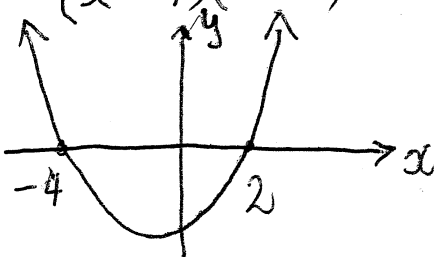
$(x-1-2y)(x-1+2y)$ (1)

(b) $\sqrt{\frac{3^{5k+2}}{3^{3k}}} = (3^{5k+2-3k})^{\frac{1}{2}}$
 $= (3^{2k+2})^{\frac{1}{2}} = 3^{k+1}$ (1)

(c) $\frac{\log a^3 + \log b^2 - (\log a + \log b^2)}{\log a^{\frac{1}{2}}}$
 $\frac{3\log a + 2\log b - \log a - 2\log b}{\frac{1}{2}\log a}$

$\frac{2\log a}{\frac{1}{2}\log a} = 4$ (1)

(d) $x^2 + 2x - 8 > 0$
 $(x+4)(x-2) > 0$



$x < -4$ and $x > 2$ (1)

(e) $|1-x| = x-1$

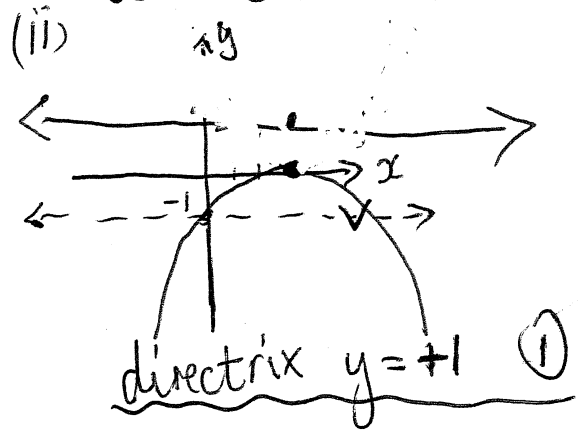
$1-x = x-1$ or $-(1-x) = x-1$
 $2 = 2x$ $-1+x = x-1$
 $x = 1$ $0 = 0$

Test $x=1$, LHS $|0|=0$ so $x=1$
 RHS $1-1=0$

(f) $(x-3)^2 = -4y$
 $(x-h)^2 = -4a(y-k)$

(i) $h=3, k=0, a=1$

$V(3,0)$ (1)



(g) $\frac{\sin \theta}{1-\cos \theta} = \frac{1+\cos \theta}{\sin \theta}$

LHS $\frac{\sin \theta}{(1-\cos \theta)} \times \frac{(1+\cos \theta)}{(1+\cos \theta)} = \frac{\sin \theta (1+\cos \theta)}{1-\cos^2 \theta}$
 $= \frac{\sin \theta (1+\cos \theta)}{\sin^2 \theta}$
 $= \frac{1+\cos \theta}{\sin \theta}$
 $= \text{RHS}$ (2)

(h) $\lim_{x \rightarrow 2} \frac{(x-2)}{(x+3)(x-2)}$

$\Rightarrow \frac{1}{5}$ (1)

But for $x > 1$,
 $|1-x| = x-1$ is also true

(2) so $x \geq 1$ is the solution.

$$\text{II (i) } \begin{aligned} 2x - y - 4 &= 0 \\ 2x + 3y - 12 &= 0 \end{aligned}$$

$$\hline -4y + 8 = 0$$

$$4y = 8 \Rightarrow y = 2$$

$$\text{So } 2x - 2 - 4 = 0$$

$$2x - 6 = 0$$

$$x = 3$$

Pt of intersection (3, 2).

$$\text{now } 2x - 3y + 1 = 0$$

$$2x + 1 = 3y$$

$$y = \frac{2}{3}x + \frac{1}{3}$$

Line \perp to this has $m = -\frac{3}{2}$.

$$\text{now } (y - y_1) = m(x - x_1)$$

$$(y - 2) = -\frac{3}{2}(x - 3)$$

$$2y - 4 = -3x + 9$$

$$\hline 3x + 2y - 13 = 0$$

$$\text{Or } y = -\frac{3}{2}x + \frac{13}{2} \quad \textcircled{1}$$

$$\hline y = -\frac{3}{2}x + 6\frac{1}{2}.$$

$$\text{II (j) } \begin{aligned} y &= 2 \sin 2x \\ y' &= 2 \times \cos 2x \times 2 \\ &= 4 \cos 2x. \end{aligned}$$

$$\begin{aligned} \text{At } x = \frac{\pi}{8}, \quad m &= 4 \times \cos \frac{\pi}{4} \\ &= 4 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{4\sqrt{2}}{2} = 2\sqrt{2}. \end{aligned}$$

using

$$(y - y_1) = m(x - x_1)$$

$$(y - \sqrt{2}) = 2\sqrt{2}(x - \frac{\pi}{8})$$

$$\hline y = 2\sqrt{2}x - \frac{\sqrt{2}\pi}{4} + \sqrt{2} \quad \textcircled{2}$$

$$(y - \sqrt{2}) = \frac{4}{\sqrt{2}}(x - \frac{\pi}{8})$$

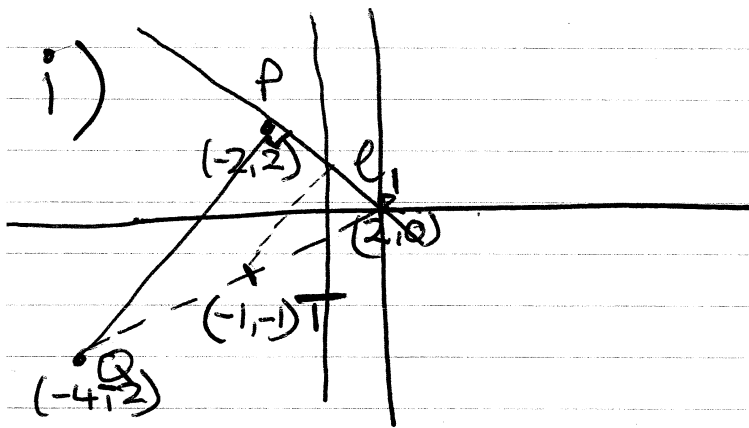
$$\sqrt{2}y - 2 = 4x - \frac{\pi}{2}$$

$$\hline 0 = 4x - \sqrt{2}y - \frac{\pi}{2} + 2.$$

$$\hline \text{or } 0 = 8x - 2\sqrt{2}y - \pi + 4$$

3 versions accepted!

QUESTION 12. 24. 2013 TRIAL.



$$M_{PQ} = \frac{2 - -2}{-2 - -4} = \frac{4}{2} = 2.$$

$$M_{l_1} = -\frac{1}{2} \quad P(-2, 2).$$

$$l_1: y - 2 = -\frac{1}{2}(x - -2).$$

$$\boxed{2y - 4 = -x - 2.}$$

$$\boxed{PR} \quad x + 2y - 2 = 0 \checkmark$$

ii) when $\boxed{y=0}$ $x=2$ $\circ \circ$ crosses at $(2, 0)$.

$$\text{iii) } MP_{QR} = \left(\frac{-4 + 2}{2}, \frac{-2 + 0}{2} \right) \\ = (-1, -1).$$

$$\text{iv) } (x + 2y - 2) = 0 \quad (-1, -1)$$

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| = \frac{1(-1) + 2(-1) - 2}{\sqrt{1^2 + 2^2}} = \frac{-5}{\sqrt{5}} = -\sqrt{5} \neq$$

$$x, -2, y \text{ geo. } -\frac{2}{x} = \frac{y}{-2} = r$$

$$-2, y, x \text{ arith. } y - (-2) = x - y = d$$

$$i) \quad xy = +4$$

$$2y - x + 2 = 0$$

$$ii) \quad \cancel{a + (a + d)}$$

$$xy = \frac{4}{x}$$

$$2\left(\frac{4}{x}\right) - x + 2 = 0$$

$$\frac{8}{x} - x + 2 = 0$$

$$8 - x^2 + 2x = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x^2 - 4x + 2x - 8) \\ x(x - 4) + 2(x - 4) \\ (x + 2)(x - 4) = 0$$

$$x = (4) \text{ or } (-2) \\ y = (1) \text{ or } (-2)$$

$$x > 0, y > 0 \text{ discard } (-2, -2)$$

$$(x - 2 + y)$$

$$(4 - 2 + 1) \cdot r = \frac{-2}{4} = -\frac{1}{2}$$

$$a = x \quad r = -\frac{2}{x}$$

$$S_{\infty} = \frac{4}{1 - (-\frac{1}{2})} = 2\frac{2}{3}$$

$$S_{\infty} = \frac{a}{1 - r} = \frac{x}{1 - (-2/x)} = \frac{x}{1 + 2/x}$$

$$\frac{x}{\frac{x+2}{x}} \div = \frac{x^2}{x+2} = S_{\infty}$$

$$\frac{8-16}{8} \\ = 2\frac{2}{3}$$

$$(x - \alpha)(x - m\alpha) \quad x^2 + px + q$$

$$mp^2 = (m+1)^2 q.$$

$$\alpha + m\alpha = -p.$$

$$\text{Rearrange } \alpha(1+m) = -p.$$

$$m\alpha^2 = q.$$

$$\therefore \alpha = \frac{-p}{1+m}.$$

$$m \left(\frac{(-p)^2}{(1+m)^2} \right) = q.$$

$$mp^2 = q(1+m)^2.$$

2 Unit - Solutions

13. (a)

(i) $y = x \sin x$

$$\frac{dy}{dx} = x \cdot \cos x + \sin x \quad \checkmark$$

(ii) $y = \ln(x^2 + 4)$

$$\frac{dy}{dx} = \frac{2x}{x^2 + 4} \quad \checkmark$$

(iii) $y = e^{5x} + x$

$$\frac{dy}{dx} = 5e^{5x} + 1 \quad \checkmark$$

(b) $f'(x) = 2x - 3$

$$f(x) = x^2 - 3x + C$$

$$f(3) = 5 \Rightarrow 5 = 9 - 9 + C \Rightarrow C = 5 \quad \checkmark$$

$$\therefore \underline{f(x) = x^2 - 3x + 5} \quad \checkmark$$

(2)

(c) (i) $\int \sqrt{x+10} \, dx$

$$= \frac{2(x+10)^{3/2}}{3} + C$$

$$= \frac{2\sqrt{(x+10)^3}}{3} + C \quad \checkmark$$

(ii) $\int_0^{\pi/8} \sec^2 2x \, dx = \left[\frac{1}{2} \tan 2x \right]_0^{\pi/8}$

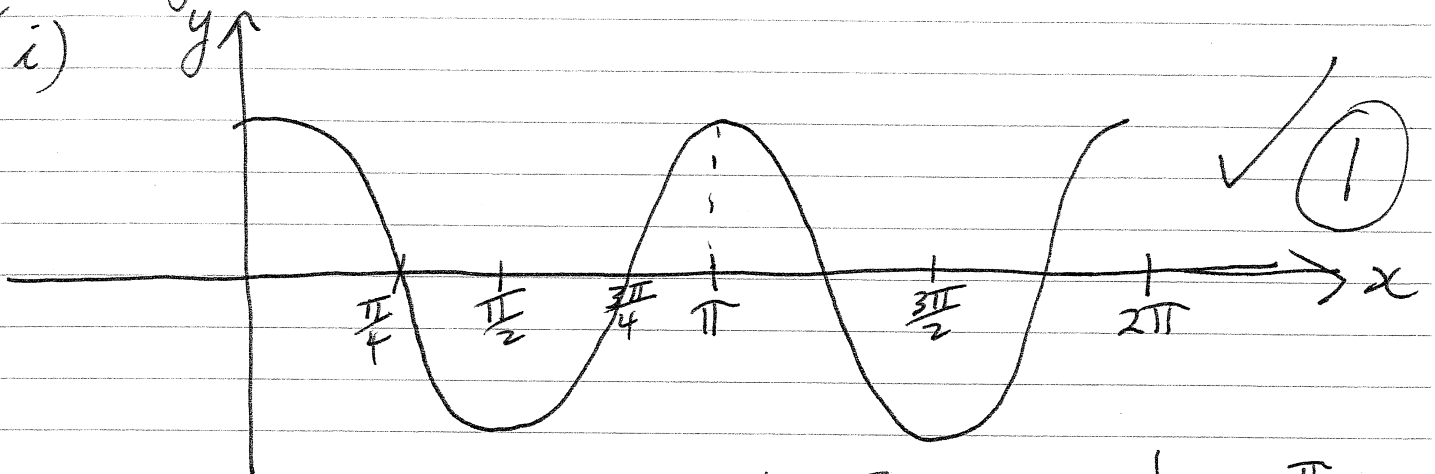
$$= \frac{1}{2} \left[\tan \frac{\pi}{4} - \tan 0 \right] = \frac{1}{2} (1 - 0)$$

$$= \frac{1}{2} \quad \checkmark$$

(2)

13(d) $y = \cos 2x$

(i)



(ii) Area = $\int_0^{\pi/4} \cos 2x \, dx + \left| \int_{\pi/4}^{3\pi/4} \cos 2x \, dx \right| + \int_{3\pi/4}^{\pi} \cos 2x \, dx$

$$= \left[\frac{\sin 2x}{2} \right]_0^{\pi/4} + \left| \left[\frac{\sin 2x}{2} \right]_{\pi/4}^{3\pi/4} \right| + \left[\frac{\sin 2x}{2} \right]_{3\pi/4}^{\pi}$$

$$= \frac{1}{2}(1-0) + \frac{1}{2}|(-1-1)| + \frac{1}{2}[0-(-1)]$$

$$= \frac{1}{2} + 1 + \frac{1}{2}$$

$$= 2 \text{ square units.}$$

(e) $P = 50000e^{-0.08t}$

(i) $t = ?$ $P = 25000 \Rightarrow 25000 = 50000e^{-0.08t}$

$$0.5 = e^{-0.08t}$$

$$\ln 0.5 = -0.08t$$

$$t = 8.66433 \text{ years.}$$

$$\therefore \underline{t = 9 \text{ years (to nearest year)}}$$

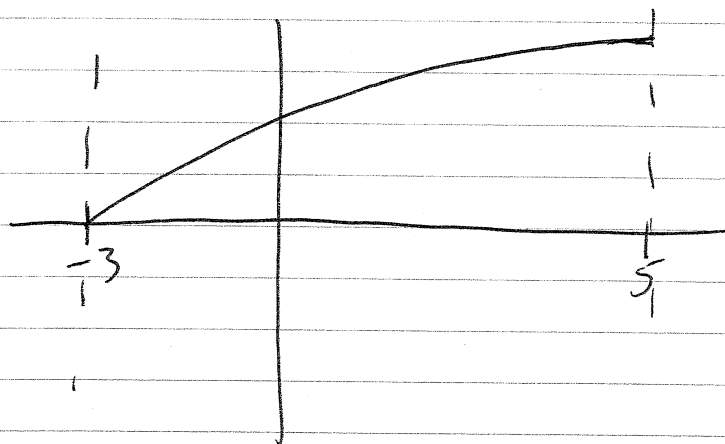
(ii) $t = 10, P = ? \Rightarrow P = 50000e^{-0.08 \times 10}$

$$= 22466 \text{ people}$$

$$t = 9, P = 50000e^{-0.08 \times 9} = 24338 \text{ people}$$

$$\therefore \text{Popn that left} = 24338 - 22466 = \underline{1872 \text{ people}}$$

13. (f) $-3 \leq x \leq 5$, $f(x) > 0$, $f'(x) > 0 \Rightarrow f(x)$ increasing
 $f''(x) < 0 \Rightarrow f(x)$ is concave down.



positive and increasing ✓
concave down ✓✓

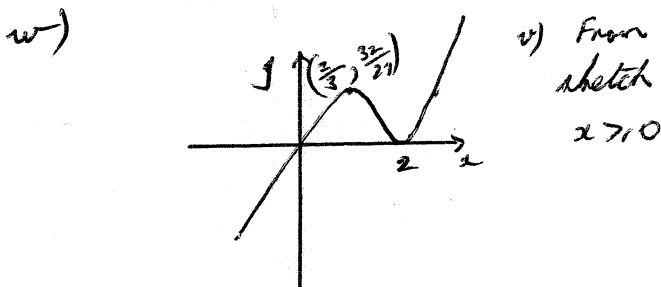
2

QUESTION FOURTEEN. MATHEMATICS 2013.

a) i) $f(x) = x(x-2)^2$
 $= x^3 - 4x^2 + 4x$
 $f'(x) = 3x^2 - 8x + 4$

ii) $f'(x) = 0$
 when $3x^2 - 8x + 4 = 0$
 $(3x-2)(x-2) = 0$
 $x = 2, y = 0$
 $x = \frac{2}{3}, y = \frac{32}{27}$

iii) $f''(x) = 6x - 8$
 $= 4$ when $x = 2$ MIN
 $= -4$ when $x = \frac{2}{3}$ MAX



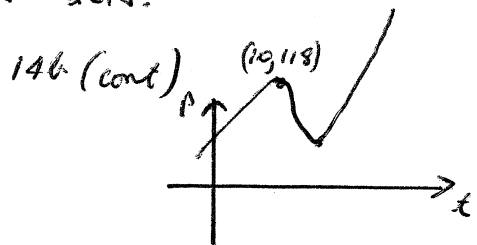
b) $P = .005t^3 - .3t^2 + 4.5t + 98$

i) When $t = 0$ $P = 98, \$98$

$\frac{dP}{dt} = .015t^2 - .6t + 4.5$
 When $t = 0$, $\frac{dP}{dt} = 4.5, \$4.50/w$

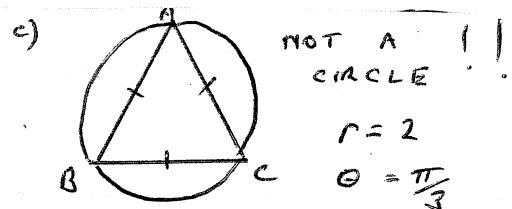
ii) $\frac{dP}{dt} = 0$ when
 $\frac{3}{200}t^2 - \frac{6}{10}t + \frac{9}{2} = 0$
 $3t^2 - 120t + 900 = 0$
 $t^2 - 40t + 300 = 0$
 $(t-10)(t-30) = 0$
 $t = 10, 30$

$\frac{d^2P}{dt^2} = .03t - 0.6$
 $= -0.3$ when $t = 10$
 Hence Relative MAX.



When $t = 10$, $P = 118$

NOTE
 A relative maximum occurs when $t = 10$ but there is no specific given domain so $P \rightarrow \infty$ as t increases
 HOWEVER
 Question does state "over the next few months" so the inference is $t = 10$ weeks.
 When $t = 10$, $P = 118, \$118$



i) Perimeter consists of 3 arcs
 $P = 3 \times \theta$
 $= 3 \times 2 \times \frac{\pi}{3}$
 $= 2\pi \text{ cm.}$

ii) Area consists of 1 triangle + 3 segments
 $A = \frac{1}{2}r^2 \sin \theta + 3 \times \frac{1}{2}r^2(\theta - \sin \theta)$
 $= \frac{3}{2}r^2\theta - r^2 \sin \theta$
 $= \frac{3}{2} \times 4 \times \frac{\pi}{3} - 4 \frac{\sqrt{3}}{2}$
 $= 2\pi - 2\sqrt{3} \text{ cm}^2$
 $= 2(\pi - \sqrt{3}) \text{ cm}^2$

Q15

$$(a) AX:XB = AY:YC \text{ (proportional division theorem)}$$

$$AR:RS = AY:YC \text{ (" " ")}$$

$$\therefore AX:XB = AR:RS$$

2

(b) (i) Amt after 1 month

$$= P \times 1.01$$

Amt after 2 months

$$= P \times 1.01^2 + P \times 1.01$$

Amt after 3 months

$$= (P \times 1.01^2 + P \times 1.01) \times 1.01 + P \times 1.01$$

$$= P (1.01^3 + 1.01^2 + 1.01)$$

 $\therefore A =$ Amt after 60 months

$$= P(1.01^{60} + 1.01^{59} + \dots + 1.01)$$

as required 2

$$(ii) A = \frac{P \times 1.01 \times (1.01^{60} - 1)}{1.01 - 1}$$

$$\therefore 40000 = P \times 1.01 \times (1.01^{60} - 1)$$

$$\therefore P = \frac{40000}{1.01(1.01^{60} - 1)}$$

$$= 484.9286 \dots$$

$$\approx \$485$$

2

$$(c) \ddot{x} = -\frac{t}{125} (30 - t)$$

$$= \frac{t^2}{125} - \frac{6t}{25}$$

$$\dot{x} = \frac{t^3}{375} - \frac{3t^2}{25} + C$$

$$\text{When } t=0, 48 = C$$

$$\therefore \dot{x} = \frac{t^3}{375} - \frac{3t^2}{25} + 48$$

$$\therefore x = \frac{t^4}{1500} - \frac{t^3}{25} + 48t + C,$$

$$\text{When } t=0: 0 = C,$$

$$\therefore x = \frac{t^4}{1500} - \frac{t^3}{25} + 48t$$

$$(i) \text{ when } t=30, \dot{x} = \frac{30^3}{375} - \frac{3 \times 30^2}{25} + 48$$

$$= 12 \text{ ms}^{-1}$$

2

$$(ii) \text{ when } t=30, x = \frac{30^4}{1500} - \frac{30^3}{25} + 48 \times 30$$

$$= 900 \text{ m}$$

2

$$(d) P(\text{at least } 10) = P(10) + P(11) + P(12)$$

$$= P(46, 55, 64) + P(56, 65) + P(66, 75)$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

2

$$(e) f(x) = x^2 \ln x - \frac{x^2}{2}$$

$$(i) f'(x) = \ln x \cdot 2x + x^2 \cdot \frac{1}{x} - \frac{2x}{2}$$

$$= 2x \ln x + x - x$$

$$= 2x \ln x$$

$$(ii) \int_1^2 x \ln x \, dx = \frac{1}{2} \int_1^2 2x \ln x \, dx$$

$$= \frac{1}{2} \left[x^2 \ln x - \frac{x^2}{2} \right]_1^2$$

$$= \frac{1}{2} \left\{ \left[4 \ln 2 - \frac{4}{2} \right] - \left[1 \cdot 0 - \frac{1}{2} \right] \right\}$$

$$= \frac{1}{2} \left\{ 4 \ln 2 - 2 + \frac{1}{2} \right\}$$

$$= 2 \ln 2 - \frac{3}{4}$$

2

Question 16 THSC 2 unit.

$$(a)(i) \angle ACB = 180 - (\alpha + \beta) \quad (\angle \text{sum } \triangle)$$

$$\begin{aligned} \angle ADC &= 360 - 90 - \alpha - \theta - (180 - (\alpha + \beta)). \\ &= 90 - \alpha - \theta + \alpha + \beta. \quad (\angle \text{sum quad}) \\ &= 90 - (\theta - \beta). \end{aligned}$$

$$(ii) \textcircled{1} \frac{q}{\sin \beta} = \frac{AC}{\sin \alpha} \quad \textcircled{2} \frac{p}{\sin \theta} = \frac{AC}{\sin D}$$

$$\textcircled{4} q = AC \frac{\sin \beta}{\sin \alpha}$$

$$AC = \frac{p \sin D}{\sin \theta}$$

$$= \frac{p \sin D \sin \beta}{\sin \theta \sin \alpha}$$

$$\begin{aligned} \textcircled{3} \sin D &= \sin(90 - (\theta - \beta)) \\ &= \cos(\theta - \beta). \end{aligned}$$

$$= \frac{p \sin \beta \cos(\theta - \beta)}{\sin \theta \sin \alpha}$$

3

(b)(i)

$$f(0) = 1 + 1 = 2$$

$$f(1) = 1 + e^2$$

$$f(2) = 1 + e^4$$

(ii)

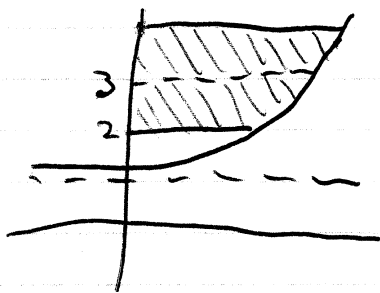
$$y = 1 + e^{2x}$$

$$e^{2x} = y - 1$$

$$2x = \ln(y - 1)$$

$$x = \frac{1}{2} \ln(y - 1)$$

(iii)



$$V = \frac{\pi}{4} \int_2^4 [\ln(y-1)]^2 dy$$

$$= \frac{\pi}{4} \times \frac{1}{3} \times \ln((\ln(1))^2 + 4(\ln(2))^2 + (\ln(3))^2)$$

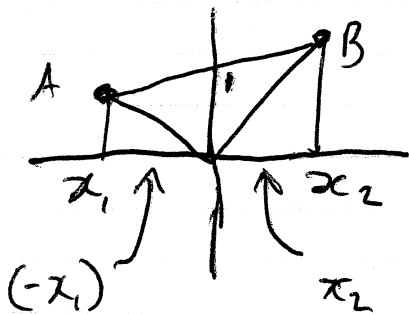
$$\approx 0.819 \text{ units}^3$$

(c)(i) $\left. \begin{array}{l} y_1 = mx_1 + 1 \quad (1) \\ x_1^2 + y_1^2 = 25 \quad (2) \end{array} \right\}$ are true since (x_1, y_1) is on both curves.

$$\begin{aligned} (1+m^2)x_1^2 + 2mx_1 - 24 &= x_1^2 + m^2x_1^2 + 2mx_1 + 1 - 25 \\ &= x_1^2 + (mx_1 + 1)^2 - 25 \\ &= x_1^2 + y_1^2 - 25 \text{ from (1)} \\ &= 25 - 25 \text{ from (2)} \\ &= 0. \end{aligned}$$

Similarly for x_2 .

(ii)



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 1 \times (-x_1) + \frac{1}{2} \times 1 \times x_2 \\ &= \frac{1}{2} (x_2 - x_1). \end{aligned}$$

3